Vision-based formation control of aerial vehicles

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***Abstract*—We propose a general solution for the problem of distributed, vision-based formation control of aerial vehicles. Our solution is based on pure bearing measurements, optionally augmented with the corresponding distances. As opposed to the state of the art, our control law does not require auxiliary distance measurements or estimators, it can be applied to leaderless or leader-based formations with arbitrary topologies, and it has global convergence guarantees. We validate our approach through simulations and experiments on a platform of three quadrotors.**

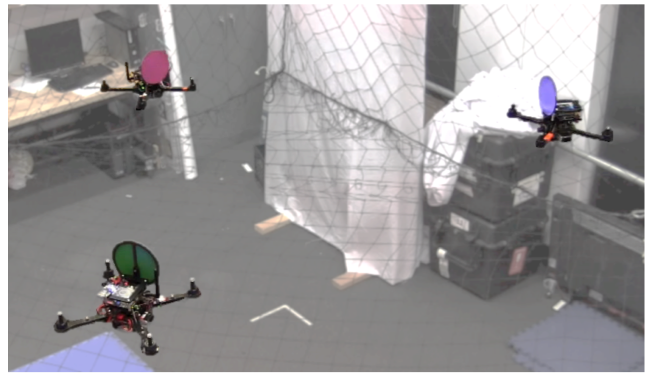

Fig. 1: A photograph of the formation of three quadrotors from our experimental setup.

I. Introduction

The goal of formation control is to move a group of agents in order to achieve and maintain a set of desired relative positions. This task has applications in many fields, such as surveillance, exploration, and transportation [10, 1, 4, 20, 11, 12]. For instance, agents with known relative positions can gather measurements which would be otherwise impossible to obtain (e.g., triangulate the position of a target), can be controlled by a single human operator, and can mimic bio-inspired energy-saving motion strategies [17].

Although the formation control problem has a long history, we focus here on vision-based solutions ([5, 14, 13]) that put more emphasis on measurements of relative *bearing* (*i.e.*, direction) as opposed to distance (which are much less reliable).

In this spirit, Bishop et al. [2] proposes a distributed control law for pure bearing formations. However, in order to be implemented, the law requires also the distance measurements corresponding to each bearing measurement. Franchi et al. [7, 6] propose different control strategies which require only one or no distance measurements, relying in turn on a special graph structure or the use of distributed estimators. This reduces the practical applicability of the method. All of these works use simple integrators to model the agent positions. Stacey and Mahony [21] consider a full dynamical model (second order) for the agents and propose an approach based on port-Hamiltonian theory. However, the derived control law again requires distance measurements. In all of the above (except [7]), it is assumed that the agents share a common rotational frame (*i.e.*, all the local reference frames can be aligned to have the same directions, but not necessarily the same origin). This could be either satisfied by using consensus-like algorithms, as in [7], or completely eschewed, as in [3, 23], although the latter are limited to triangular and 2-D circular formation topologies.

Our contributions. In this work, we keep the assumptions of a common rotational frame and a simple integrator model. However, we propose a flexible framework which, unlike previous work, has global convergence guarantees (Theorem 1) and requires only minimal assumptions on the formation (i.e., it should be rigid, as defined later). The proposed control law is computed directly from bearing measurements (optionally augmented with distance measurements), it does not need any auxiliary estimator, and it is distributed (in fact, if each agent can sense all of its neighbors, no communication is necessary). In addition, for particular cases, our control law becomes *correspondence-less*, in the sense that no correspondence between measured and computed bearings is required. Our approach also naturally covers both 2-D and 3-D formations, and can be (optionally) used with leader nodes. No existing works has all these properties at the same time. Finally, we validate our solution through simulations and a set of three quadrotors equipped with on-board cameras.

II. Notation and definitions

We identify the set of N agents as $V = \{1, \dots, N\}$, and their location as $\{x_i\}_{i \in V}$, $x_i \in \mathbb{R}^n$. We define the *distance* between nodes $i, j \in V$ as

$$d_{ij}(x_i, x_j) = \|x_j - x_i\|, \tag{1}$$

and, when $d_{ij} \neq 0$, the *bearing direction* as

$$\beta_{ij}(x_i, x_j) = d_{ij}^{-1}(x_j - x_i). \tag{2}$$

We define a *bearing+distance formation* as a pair $(\mathcal{F}, \boldsymbol{x})$, where:

- $\mathcal{F} = (V, E_b, E_d)$ is a double graph in which $E_b \subseteq V \times V$ (resp., $E_d \subseteq E_b$) contains the set of pairs (i, j) for which agent i can measure the bearing β_{ij} (resp., the range d_{ij}).
- $\boldsymbol{x} = \text{stack}(\{x_i\}_{i \in V})$ is the *configuration* of the formation.

We assume that E_b and E_d are symmetric, *i.e.*, if $(i, j) \in E_b$, then also $(j, i) \in E_b$ (the same for E_d). Note that, since $\beta_{ij} =$

$-\beta_{ji}$ and $d_{ij} = d_{ji}$, this can either happen naturally or through communication (if node i senses j, it can send its measurements to j). We stress that $E_d \subseteq E_b$, *i.e.*, that agent i can measure the distance d_{ij} only if it can also measure the bearing β_{ij} (this is a natural assumption for vision measurements). We generally assume $E_d \neq \emptyset$. For the particular case $E_d = \emptyset$ (*i.e.*, no range measurements) we call $\mathcal{F}$ a *pure bearing formation*.

We denote the vector of all available measurements as

$$\boldsymbol{y}(\boldsymbol{x}) = \operatorname{stack}(\{\beta_{ij}\}_{(i,j)\in E_b}, \{d_{ij}\}_{(i,j)\in E_d}). \tag{3}$$

Two formations $(\mathcal{F}, \boldsymbol{x})$ and $(\mathcal{F}, \boldsymbol{x}')$ with the same rotational frame of reference are said to be

- *equivalent* if $\boldsymbol{y}(\boldsymbol{x}) = \boldsymbol{y}(\boldsymbol{x}')$,
- *similar* if $\boldsymbol{x}'$ can be obtained from $\boldsymbol{x}$ using a translation followed by a dilation,
- *congruent* if $\boldsymbol{x}'$ can be obtained from $\boldsymbol{x}$ using a translation.

If the rotational reference frames of two formations are not the same, then the definitions above need to be extended to compensate for the difference. A bearing+distance (resp., pure bearing) formation is said to be *rigid* (resp., *parallely rigid*, or simply *rigid*) if all formations which are equivalent to it are also congruent (resp., similar). In practice, one can check whether a formation is rigid by checking the rank of the so called *rigidity matrix* [2, 19]. Intuitively, a formation is rigid when the configurations $\boldsymbol{x}$ that produce the same measurements $\boldsymbol{y}$ have also the same "shape". In practice, the translation ambiguity is fixed either by the position of the leader or through the centroid invariance (see Section III-D).

Finally, we call a formation *leaderless* if every agent in the network is autonomous, and *leader-based* if one of the agents (say, node $\hat{\imath}$) follows an independently specified trajectory.

III. Formation control

We assume that each agent follows a simple integrator model:

$$\dot{x}_i(t) = u_i, \tag{4}$$

where u_i is a control input. Given desired measurements $\boldsymbol{y}_g$ which are consistent with a desired rigid formation $(\mathcal{F}, \boldsymbol{x}_g)$, our goal is to design inputs u_i that drive the agents into a configuration similar (for pure-bearing formations) or congruent (for bearing+distance formations) to $\boldsymbol{x}_g$. We propose to use the negative gradient of a special cost function $\varphi(\boldsymbol{x})$, *i.e.*,

$$u_i = -\operatorname{grad}_{x_i} \varphi(\boldsymbol{x}). \tag{5}$$

Of course, if present, the leader will follow an independent control law $u_{\hat{\imath}}$ (for the analysis, we will assume $u_{\hat{\imath}} \equiv 0$, although, with sufficiently high gains, in practice our control law can satisfactorily track any smooth trajectory). We will now introduce the proposed control law and its properties.

A. The cost function

The cost function we propose is of the following form:

$$\varphi(\boldsymbol{x}) = \alpha_b \sum_{(i,j)\in E_b} \varphi^b_{ij}(x_i, x_j) + \alpha_d \sum_{(i,j)\in E_d} \varphi^d_{ij}(x_i, x_j), \tag{6}$$

$$\varphi^b_{ij}(x_i, x_j) = d_{ij} f_b\big(c_{ij}\big), \tag{7}$$

$$\varphi^d_{ij}(x_i, x_j) = f_d\big(q_{ij}\big). \tag{8}$$

We now proceed to explain the various parts of this equation. At a high level, φ is composed of a summation, with positive weights α_b and α_d, over the edges E_b and E_d, where each term is a function of one of the two following "similarity measures" between the current and desired measurements, $\boldsymbol{y}(\boldsymbol{x})$ and $\boldsymbol{y}_g$:

$$c_{ij}(x_i, x_j) = \beta^T_{g,ij}\beta_{ij}, \qquad (i,j) \in E_b \tag{9}$$

$$\begin{aligned} q_{ij}(x_i, x_j) &= \beta^T_{g,ij}\big(x_j - x_i - (x_{g,j} - x_{g,i})\big) \\ &= \beta^T_{g,ij}(d_{ij}\beta_{ij} - d_{g,ij}\beta_{g,ij}) \\ &= d_{ij}c_{ij} - d_{g,ij}, \qquad (i,j) \in E_d. \end{aligned} \tag{10}$$

Eq. (9) is the cosine of the angle between the measured and desired bearings, while (10) quantifies the discrepancy between the measured and desired relative position of the agents projected on the line given by $\beta_{g,ij}$ ($c_{ij} = 1$ and $q_{ij} = 0$ when bearing and distances coincide). We use q_{ij} instead of a simple difference of the distances because q_{ij} is actually linear in the configuration $\boldsymbol{x}$ (as it can be seen from the first equality in (10)). Each of these similarities is weighted by a reshaping function (f_b or f_d), for which we will require some properties in order to show convergence. For the experimental validation, we will use $f_b(c) = 1 - c$ and $f_d(q) = \frac{1}{2}q^2$.

B. Global asymptotic stability

In this section we state a result on the convergence of our control law to the desired formation from any initial condition.

Theorem 1. *Assume that the desired formation $(\mathcal{F}, \boldsymbol{x}_g)$ is rigid, and that, for leader-based formations, the leader agent is stationary. Assume also that*

$$f_b(c), f_d(q) \geq 0, \textit{ with equality iff } c = 1, q = 0 \tag{11}$$

$$\frac{\mathrm{d}f_b(c)}{\mathrm{d}c} \begin{cases} \leq 0 & \textit{and finite for } c = 1, \\ < 0 & \textit{otherwise}, \end{cases} \tag{12}$$

$$f_b(c) + (1-c)\frac{\mathrm{d}f_b(c)}{\mathrm{d}c} \leq 0, \tag{13}$$

$$\operatorname{sign}\left(\frac{\mathrm{d}f_d(q)}{\mathrm{d}q}\right) = \operatorname{sign}(q). \tag{14}$$

Then, a configuration $\boldsymbol{x}$ is a global minimizer and a critical point of φ if and only if it is equivalent to $\boldsymbol{x}_g$. It follows that any trajectory defined by (5) *converges to a configuration which is congruent (or, for pure bearing formation, similar) to the desired one, $\boldsymbol{x}_g$.*

At a high level, this result can be proved by first showing that $\boldsymbol{y}(\boldsymbol{x}) = \boldsymbol{y}_g$ if and only if $\varphi(\boldsymbol{x}) = 0$. Then, one can show that the derivative of φ along lines starting from a global minimizer never vanishes, and hence no other critical points are present. This, together with some rather standard technical remarks on gradient systems, implies global asymptotic stability.

C. Correspondence-less control law

For the particular choice $f_b(c) = 1 - c$ and with pure bearing formations, our control law simply becomes:

$$u_i = -\alpha_b \left(\sum_{j:(i,j)\in E_b} \beta_{g,ij} - \sum_{j:(i,j)\in E_b} \beta_{ij} \right). \tag{15}$$

Note that (15) can be separated in (unordered) sums containing either the current or the desired measurements, and no specific correspondences between the two are needed. As a result, we say that the law is *correspondence-less*.

D. Centroid invariance

Notice that, for leaderless formations, we have $\operatorname{grad}_{x_i} \varphi^b_{ij}(x_i, x_j) = -\operatorname{grad}_{x_j} \varphi^b_{ji}(x_j, x_i)$ (similarly for φ^d_{ij}). Then, if we let the centroid of the formation be

$$m = \frac{1}{N} \sum_{i \in V} x_i, \tag{16}$$

it follows that m is invariant with respect to the trajectories of the closed loop system, *i.e.*, $\dot{m} = 0$. This ensures that, even without a leader, the formation as a whole will not drift from its initial position.

IV. Simulations

We first validate the proposed controller through a simulated network of seven agents. The desired leader-based formation is one where all the agents are equally spaced around a circle. Each agent i measures its bearing with respect to agent $i+1$ and $i+2$ (and viceversa), $i = 1, \dots, 5$. We also add a single distance measurement. The initial positions of the agents are random, and the leader ($\hat{\imath} = 4$) moves with constant velocity on the negative x-axis and with a sinusoidal motion on the y-axis. Figure 2 shows that the agents quickly converge to the desired formation and then closely follow the desired trajectory.

V. Experiments

The presented approach is tested using 3 Asctec Hummingbird quadrotors [8]. Each vehicle is equipped with an ODROID-XU [15] computer board with Ubuntu Arm 13.04, ROS [18], and OpenCV [16]. Additional hardware includes a monocular RGB camera from Matrix-Vision [9], a 13 cm diameter colored circle for vision detection, and retroreflective markers that can be tracked by Vicon cameras [22] for velocity feedback. Figure 3 shows a block diagram of the system.

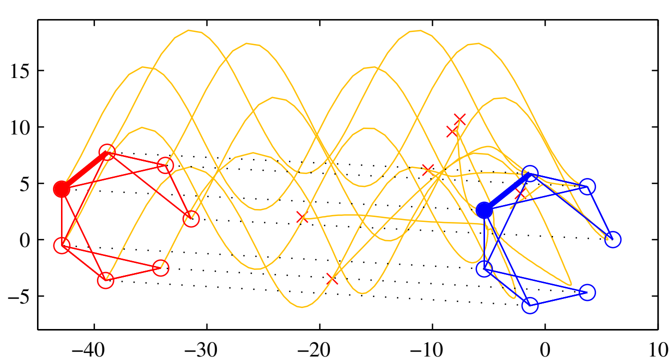

Fig. 2: Simulation of bearing+distance formation with a moving leader (solid circle). Blue: desired formation. Red: final formation. Red crosses: initial configuration of the agents. Thick lines: edge with distance measurement.

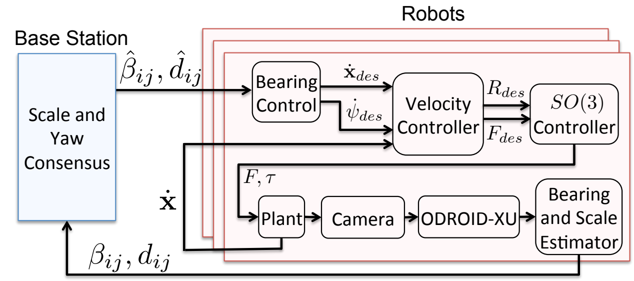

Fig. 3: This figure displays a block diagram of the multi-robot system. Using the consensus algorithm, the relative yaw and scale are estimated and used to compute a desired velocity, $\dot{\mathbf{x}}_{des}$, for each of the robots using the proposed control alogorithm.

Each vehicle is configured to visually detect the colored circular identifiers on the other robots. We estimate the bearing and distance by fitting an ellipse to the contour points of the other robots targets'. The Inertial Measurement Unit (IMU) on each robot is used to rotate the bearings to a level plane and are are sufficient to drive the control algorithm discussed in Section III. In practice, to improve performance, we use a robust consensus algorithm running on a ground station to collect the measurements from all robots and estimatea common yaw and scale that give a reliable estimation for all d_{ij} (with a fully connected graph, this uses $N(N-1)$ sets of measurements).

To validate the proposed control approach, we design and execute an experiment with gross 3-D motion of the formation. The formation's global position and orientation is specified by directly controlling one robot, designated as the leader, using position feedback from the external motion capture system. The other robots in the formation maintain their relative positions to this lead vehicle and rely on the external motion capture system only for velocity feedback. The vision algorithm, the velocity controller, and the position controller run onboard each of the vehicles, with the vision loop executed at 15 Hz.

In the presented experiment, the leader is commanded to translate in both horizontal and vertical directions, and the other two robots follow the motion (see Figure 4).

VI. Conclusions

In this work, we proposed a framework for bearing-based formation control that can be naturally complemented with inter-agent distance measurements and leader agents. We also proposed a correspondence-less version of the control law and shown global convergence and centroid invariance. We tested our approach in simulated and real experiments. In the future, we plan to: investigate more rigorously the effect of noise and of moving or changing formations on the control law, extend the approach to second-order agents (which can be more closely mapped to real quadrotors, thus eliminating the need of external localization systems), and take into account other physical constraints such as limited field of views.

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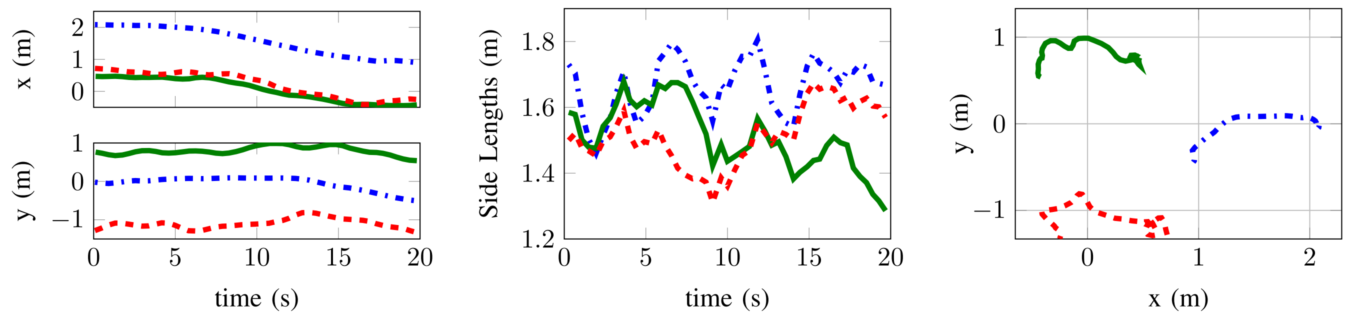

Fig. 4: Experimental results for a moving formation of three quadrotors. Blue line: leader. Other lines: followers. Left plot: robots' coordinates versus time. Middle plot: side lengths of the triangle as estimated using consensus (the colors of the lines correspond to the side of the triangle opposite that robot, e.g., the blue line is the side length between the two followers). Right plot: projection of the motion onto the $x - y$ plane (the robots start at the rightmost position of their trajectories).

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